Chapter 13 Problems Plus #1

Each edge of a cubical box has length 1m. The box contains nine spherical balls of radius *R*. The center of one ball is at the center of the cube and touches each of the other eight balls, each of which touch thre­e­­ sides of the box. Find *R*.

Suppose the coordinates of opposite corners of a cube in 3d space are and So the center of the cube is and the length of the diagonal is . Because we are dealing with a cube, we can say that , so let’s call that quantity *s*. With this, our equation for the length of the diagonal becomes .

At one of these corners, there lies a sphere of radius *r* that “touches” the “sides” of the cube at a distance *R* from the center of the sphere. Thus, we let the center of the sphere have a center , and the length of the line from the corner to the center of the sphere is . This sphere touches another sphere whose center is *C* and the distance between the center of the centers of the spheres is . Notice that their point of intersection lies upon the diagonal of the cube. This gives the expression,

* , and , so

Thus, the radius *R* of the spheres for a box of side length 1 is equal to